Exercise 68

The equation $y'' + y' - 2y = x^2$ is called a **differential equation** because it involves an unknown function y and its derivatives y' and y''. Find constants A, B, and C such that the function $y = Ax^2 + Bx + C$ satisfies this equation. (Differential equations will be studied in detail in Chapter 9.)

Solution

To determine the constants, plug in the function $y = Ax^2 + Bx + C$ into the differential equation.

$$y'' + y' - 2y = x^2$$

$$(Ax^2 + Bx + C)'' + (Ax^2 + Bx + C)' - 2(Ax^2 + Bx + C) = x^2$$

Calculate each derivative.

$$(2Ax + B)' + (2Ax + B) - 2(Ax^2 + Bx + C) = x^2$$

Take the last derivative.

$$(2A) + (2Ax + B) - 2(Ax^2 + Bx + C) = x^2$$

Simplify the left side.

$$-2Ax^2 + (2A - 2B)x + (2A + B - 2C) = 0 + 0x + x^2$$

Match the coefficients of x on both sides to obtain a system of equations for A, B, and C.

$$-2A = 1$$

$$2A - 2B = 0$$

$$2A + B - 2C = 0$$

Solving it yields

$$A = -\frac{1}{2} \quad \text{and} \quad B = -\frac{1}{2} \quad \text{and} \quad C = -\frac{3}{4}.$$

Therefore,

$$y = -\frac{1}{2}x^2 - \frac{1}{2}x - \frac{3}{4}$$

satisfies the differential equation.